MATHEMATICS

Unit 1: Real Analysis

Finite-countable and uncountable sets,-Bounded and unbounded sets-Archimedean Property-Ordered field-Completeness of **R-**Extended real number system-Sequences and series-limsup and liminf of a sequence-convergence of sequences and series-uniform convergence-continuity of a function-types of discontinuities-uniform continuity-differentiability-Roll's theorem-mean value theorem –monotone functions, functions of bounded variations,-Riemann Integral and its properties-Improper integrals and their convergence and uniform convergence- sequence of functions and series of functions-point wise convergence and uniform convergence-Bolzano-Weierstrass Theorem-Compact subsets of $\mathbf{R}^n$-Heine-Borel Theorem-Riemann-Stieltjes integral and its properties- partial, directional and total derivatives in $\mathbf{R}^n$.

Unit 2: Complex Analysis

Algebra of complex numbers, Riemann Sphere, Stereographic projection, lines, circles, cross ratio, Mobious transformation, Analytic functions, Cauchy-Riemann equations, line integrals, Cauchy's theorem for convex regions, Morera's theorem, Liouville's theorem, Fundamental Theorem of Algebra, Cauchy's Integral formula, power series representation, classification of singularities, Riemann theorem for removable singularities, Taylor's and Laurent's series expansions, maximum modulus principle. Schwarz lemma, Open mapping theorem, Contour integration, Conformal mapping, Entire functions, Harmonic functions, Elliptic functions, Analytic continuation.

3. Algebra

Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation groups, symmetric groups, alternating groups, simple groups, Sylow's theorem, Finite abelian groups, Rings, ideals, integral domains, polynomial rings, Euclidean ring, Principal ideal domains, Unique factorisation domains, Finite fields, Extension fields, Splitting fields, Galois Theorem.

Vector spaces, linear independence, bases, dimension, subspaces, quotient spaces, algebra of linear transformations, kernel, range, isomorphism, matrix representation of a linear transformation, change of bases, Dual bases, dual space, projection, transpose, trace, determinant, Hermitian, Unitary and normal transformations, eigen values and eigen vectors, Cayley-Hamilton theorem, Invariant subspaces, canonical forms: triangular form, Jordan form, rational canonical form.

4. Topology

Topological spaces-Basis for a topology-The product topology –The subspace topology-Closed sets and limit points, Continuous functions-the product topology-The metric topology. Connected spaces-connected subspaces of the Real line –Components and local connectedness, Compact spaces-compact subspaces of the Real line– Limit Point Compactness – Local Compactness. The Countability Axioms – The separation Axioms – Normal spaces – The Urysohn Lemma –The Tietze extension theorem.

5. Measure Theory and Functional Analysis

Measure Theory: Lebesgue Outer Measure-Measurable sets-Regularity – Measurable Functions-Borel and Lebesgue Measurablility- Integration of Non-negative functions – The General Integral – Riemann and Lebesgue Integrals, Field of sets, sigma-field of sets, finitely additive set function and countably additive set function, measure, Measurable and measure spaces, Extension of measures, signed measures, Jordan, Hahn decomposition theorem, Monotone convergence Theorem, Fatou's lemma, Dominated convergence theorem, absolute continuity, $\mathbf{L}^p$ spaces-Convex functions, Jensen's inequality, Holder's and Minkowski's inequalities, Radon-Nikodym derivative, Fubini's Theorem.

Functional Analysis: Normed Linear space-Continuous Linear Transformations – Banach spaces-The Hahn-Banach Theorem – The natural embedding of N in N** - Open mapping theorem – Closed graph Theorem – Uniform boundedness theorem – conjugate of an operator – Inner product space – Orthogonalisation process – Hilbert Space – Orthonormal sets-Orthogonal complements – conjugate space H* - Adjoint of an operator – Self-adjoint – Normal and Unitary Operators –Projections.

6. Differential Equations

Ordinary Differential Equations : Second order homogeneous equations-Initial value problems-Linear dependence and independence-Wronskian and a formula for Wronskian-Non-homogeneous equation of order two. Homogeneous and non-homogeneous equation of order n – Initial value problems- Annihilator method to solve non-homogeneous equation- Algebra of constant coefficient operators. Initial value problems – Existence and uniqueness theorems- Solutions to solve a non-homogeneous equation – Wronskian and linear dependence – reduction of the order of a homogeneous equation - homogeneous equation with analytic coefficients – The Legendre equation. Euler equation – Second order equations with regular singular points – Exceptional cases – Bessel Function. Equation with variable separated – Exact equation- method of successive approximations – the Lipschitz condition- convergence of the successive approximations and the existence theorem.

Partial Differential Equations: Linear and non-linear first order partial differential equations – Second order equations in two independent variables – canonical forms – equations with constant coefficients - general solution. The Cauchy problem – Homogeneous wave equation – Initial Boundary value problem -Non- homogeneous boundary conditions – Non-homogeneous wave equation – Riemann method – Goursat problem – spherical wave equation – cylindrical wave equation. Separation of variable – Vibrating string problem – Existence and uniqueness of solution of vibrating string problem. – Heat conduction problem – Existence and uniqueness of solution of heat conduction problem – Laplace and beam equations. Boundary value problems – Maximum and minimum principles – Uniqueness and continuity theorem – Dirichlet Problem for a circle, a circular annulus, a rectangle – Dirichelt problem involving Poisson equation – Neumann problem for a circle and a rectangle. The Delta function – Green's function – Method of Green's function – Dirichlet Problem for the Laplace and Helmholtz operators.

7. Mechanics and Numerical Methods

Mechanics : Generalised coordinates - Constraints – Virtual work- Energy and Momentum, Derivation of Lagrange's equations-Examples –Integrals of motion. Hamilton's Principle – Hamilton's Equation – Other variational principle. Hamilton Principle function – Hamilton-Jacobi Equation- Separability, Differential forms and generating functions – Special Transformations-Lagrange and Poisson brackets.

Numerical Methods: Representation of numbers (binary, octal, decimal, hexadecimal) – Errors – Difference Table – Difference formula – Solution of non-linear equations: Bisection, secant, regula-falsi, Newton-Raphson, Fixed iteration. Solution of system of equations: Gauss Elimination, Jacobi, Gauss-Jordan, Gauss-Seidal, LU decomposition. Solution of ordinary differential equations: Taylor Series, Euler and modified Euler, Runge-Kutta method of order two and four, Milne-Simpson, Adams-Badsforth method.

8. Probability and Mathematical Statistics

Probability : Random events – Probability axioms – Combinatorial formulae – conditional probability-Bayes Theorem – Independent events – Random Variables – Distribution Function – Joint Distribution – Marginal Distribution – Conditional Distribution – Independent random variables – Function of random variables. Expectation – Moments – The Chebyshev Inequality – Absolute moments Cumulant Generating Function, Moment Generating Function and Probability Generating function – Properties of characteristic functions – Characteristic functions and moments – characteristic function of the sum of the independent random variables – Determination of distribution function by the Characteristic function - Probability generating functions. One point, two point, Binomial – Poisson distribution – Uniform (discrete and continuous) – normal-gamma distributions. Weak law of large numbers – Central limit theorem (Lindberg Theorem and Lapunov Theorem) Borel-Cantelli Lemma – Kolmogorov Strong Law of large numbers.

Mathematical Statistics: Sampling: Sample mean, sample variance and their independence-Moments of sample mean and sample variance, t distribution, F distribution. Point Estimation : Unbiasedness, consistency, sufficiency, efficient and asymptotically most efficient-Method of moments: One parameter and two parameters cases-Maximum likelihood Estimation: One parameter and two parameter cases unbiasedness, mean square error, CR bound. Interval Estimation: Derivation of confidence interval:-The pivotal method, confidence limits, sample size, confidence interval for the normal distribution (mean, variance)-Confidence interval for Binomial and Poisson-Confidence interval for two sample problems (Two normal means, two population variances, two population propositions, two Poisson parameters, paired data). Hypotheses, test statistics, decision and errors: Hypotheses (Null, alternative, simple and composite), one sided and two sided tests, test statistics, errors (Type I and II errors), Best Test (smallest type-II error), p –values. Best Tests: Testing the value of a population mean, of population variance, of population proposition. Of the mean of Poisson. Best Tests: Testing the value of the difference between two population means, ratio of two population variances, difference between population propositions, difference between two Poisson means, paired data. Tests and confidence intervals: chi-square test, goodness of fit, contingency table for independence. ANOVA: One way and two way classifications.

9. Differential Geometry and Graph Theory

Space curve - Arc length – tangent – normal and binormal – curvature and torsion – contact between curves and surfaces- tangent surface – involutes and evolutes- Intrinsic equations – Fundamental Existence Theorem for space curves-Helics. Surface – curves on a surface – Surface of revolution – Helicoids – Metric – Direction coefficients – families of curves-Isometric correspondence- Intrinsic properties. Geodesics – Canonical geodesic equations – Normal property of geodesics- Existence Theorems – Geodesic parallels –Geodesics curvatures – Gauss-Bonnet Theorem –Gaussian curvature-surface of constant curvature. The second fundamental form-Principle curvature – Lines of curvature – Developable – Developable associated with space curves and with curves on surface – Minimal surfaces – Ruled surfaces. Compact surfaces whose points are umblics-Hilbert's lemma – Compact surface of constant curvature – Complete surface and their characterization – Hilbert's Theorem – Conjugate points on geodesics.

Graph Theory: Graphs and simple graphs – Graphs Isomorphism – The Incidence and Adjacency Matrices – Subgraphs – Vertex Degrees – Paths and Connections – Cycles – Trees – Cut Edges and Bonds – Cut Vertices. Connectivity - Blocks – Euler tours- Hamilton Cycles. Matchings – Matchings and Coverings in Bipartite Graphs – Edge Chromatic Number. Vizing's Theorem. Independent sets – Ramsey's Theorem – Chromatic Number – Brooks' Theorem – Chromatic Polynomials. Plane and planer Graphs – Dual graphs – Euler's Formula – The Five – Colour Theorem and the Four-Colour Conjecture-Directed graphs.

10. Mathematical Programming and Fluid Dynamics

Convex sets-Hyperplane-Open and closed half spaces-Formulation of Linear Programming problem – Graphical solution – Types of solutions – Simplex procedure – method of penalty – Two – phase technique-special cases in simplex method applications – Duality – Economic Interpretation of duality- Dual Simplex method – Generalised simplex Tableau in Matrix Form – Efficient Computational algorithm – Transportation and Assignment problems as linear programming problems.

Fluid Dynamics : Real fluids and Ideal fluids- Velocity of a fluid at a point, Stream lines, path lines, steady and unsteady flows- Velocity potential – The vorticity vector – Local and particle rates of changes – Equations of continuity- Acceleration of a fluid – Conditions at a rigid boundary. Pressure at a point in a fluid at rest – Pressure at a point in a moving fluid - Conditions at a boundary of two inviscid immiscible fluids – Euler's equation of motion – Steady motion under conservative body forces. Sources, sinks and doublets – Images in a rigid infinite plane - Axis symmetric flows – Stokes stream function-Two dimensional flow – The stream function – The complex potential for two dimensional, irrotational in compressible flow- Complex velocity potentials for standard two dimensional flows – Two dimensional Image systems – The Milne Thomson circle Theorem. Stress components in a real fluid. Relations between Cartesian components of stress – Translational motion of fluid elements – The rate of strain quadric and principle stresses – Some further properties of the rate of strain quadric - Stress analysis in fluid motion – Relation between stress and rate of strain – The coefficient of viscosity and Laminar flow – The Navier – Stokes equations of motion of a Viscous fluid.