MATHEMATICS

UNIT 1: REAL ANALYSIS

Ordered sets – Fields – Real field – The extended real number system – The complex field- Euclidean space - Finite, Countable and uncountable sets - Limits of functions - Continuous functions – Continuity and compactness – Continuity and connectedness – Discontinuities - Monotonic functions - Equi-continuous families of functions, Stone – Weierstrass theorem – Cauchy sequences – Some special sequences – Series - Series of nonnegative terms – The number e – The root and ratio tests – Power series – Summation by parts – Absolute convergence - Addition and multiplication of series - Rearrangements, The Derivative of a Real Function – Mean Value Theorem - The Continuity of Derivatives - L'Hospital's Rule – Derivatives of Higher Order - Taylor's Theorem – Differentiation of Vector valued functions – Some Special Functions - Power Series – The Exponential and Logarithmic functions – The Trigonometric functions - The algebraic completeness of the complex field – Fourier series – The Gamma function - The Riemann – Stieltjes Integral – Definition and Existence of the Integral – Properties of the Integral - Integration and Differentiation – Integration of Vector – valued functions – Rectifiable curves.

UNIT 2: COMPLEX ANALYSIS

Spherical representation of complex numbers – Analytic functions – Limits and continuity - Analytic Functions – Polynomials – Rational functions – Elementary Theory of Power series-Sequences – Series – Uniform Convergence – Power series - Abel's limit functions – Exponential and Trigonometric functions – Periodicity – The Logarithm - Analytical Functions as Mappings - Conformality - Arcs and closed curves - Analytic functions in Regions – Conformal mapping - Length and area - Linear transformations - Linear group – Cross ratio – symmetry - Oriented Circles – Families of circles – Elementary conformal mappings – Use of level curves – Survey of Elementary mappings – Elementary Riemann surfaces – Complex Integration - Fundamental Theorems - Line Integrals – Rectifiable Arcs - Line Integrals as Arcs- Cauchy's Theorem for a rectangle and in a disk-Cauchy's Integral Formula - Index of point with respect to a closed curve - The Integral formula - Higher order derivatives – Local properties of analytic functions - Taylor's Theorem – Zeros and Poles – Local mapping – Maximum Principle – The General form of Cauchy's Theorem – Chains and Cycles – Simple connectivity Homology – General statement

of Cauchy's theorem – Proof of Cauchy's theorem – Locally exact differentials - Multiply connected regions – Calculus of residues - Residue Theorem – Argument Principle - Evaluation of definite Integrals – Harmonic Functions – Definition and basic properties – Mean - value Property - Poisson's formula - Schwarz's Theorem – Reflection Principle – Weierstrass's theorem – Taylor's series – Laurent series.

UNIT 3: ALGEBRA

Another counting principle - Sylow's theorems – Direct products – Finite abelian groups, Polynomial rings – Polynomials over the rational field – Polynomial rings over commutative rings - Extension fields – Roots of polynomials – More about roots – The element of Galois theory – Finite fields - Wedderbum's theorem on finite division rings – Theorem of Frobenius – The algebra of polynomials - Lagrange Interpolation – Polynomial ideals – The prime factorization of a polynomial –Commutative rings – Determinant functions – Permutations and the uniqueness of determinant - Classical adjoint of a matrix – Inverse of an invertible matrix using determinants - Characteristic values – Annihilating polynomial – Invariant subspaces – Simultaneous triangulation –Simultaneous diagonalization – Direct sum decompositions – Vector spaces Bases and dimension Subspaces – Matrices and linear maps – Rank nullity theorem – Inner product spaces – Orthonormal basis – Gram – Schmidt orthonormalization process – Eigen spaces – Algebraic and Geometric multiplicities – Cayley – Hamilton theorem – Diagonalization – Direct sum decomposition - Invariant direct sums – Primary decomposition theorem – Unitary matrices and their properties - Rotation matrices - Schur, Diagonal and Hessenberg forms and Schur decomposition - Diagonal and the general cases – Similarity Transformations and change of basis – Generalised eigen vectors - Canonical basis – Jordan canonical form – Applications to linear differential equations -Diagonal and the general cases - An error correcting code – The method of least squares – Particular solutions of non-homogeneous differential equations with constant coefficients - The Scrambler transformation.

UNIT 4: TOPOLOGY

Topological spaces – Basis for a topology – Product topology on finite Cartesian products –Subspace topology – Closed sets and Limit points – Continuous functions - Homeomorphism - Metric Topology – Uniform limit theorem – Connected spaces – Components – Path components - Compact spaces – Limit point compactness -

Local compactness – Countability axioms -T1-spaces – Hausdorff spaces - Completely regular spaces – Normal spaces – Urysohn lemma - Urysohn metrization theorem - Imbedding theorem - Tietze extension theorem - Tychonoff theorem.

UNIT 5: MEASURE THEORY AND FUNCTIONAL ANALYSIS

MEASURE THEORY : Lebesgue Outer Measure – Measurable Sets – Regularity – Measurable Functions – Boreland Lebesgue Measurablity – Abstract Measure - Outer Measure – Extension of a Measure – Completion of a Measure – Integrals of simple functions – Integrals of Non Negative Functions – The Generallntegral – Integratiion of Series – Riemann and Lebesgue Integrals – Legesgue Differentiation Theorem – Integration and Differentiation – The Lebesgue Set – Integration with respect to a general measure Convergence in Measure – Almost Uniform convergence - Signed measures and Hahn Decomposition - Radon-Nikodym Theorem and its applications- Measurability in a product space – The Product measure and Fubini's Theorem.

FUNCTIONAL ANALYSIS: Banach spaces – Continuous linear transformations - The Hahn-Banach theorem – The natural imbedding of N in N** - The open mapping theorem - Closed graph theorem - The conjugate of an operator – Uniform boundedness theorem - Hilbert Spaces – Schwarz inequality – Orthogonal complements – Orthonormal sets - Bessel's Inequality – Gram – Schmidt orthogonalization process – The conjugate space H^*- Riesz representation theorem – The adjoint of an operator - Self-adjoint operators – Normal and unitary operators – Projections – Matrices – Determinants and the spectrum of an operator - spectral theorem – Fixed point theorems and some applications to analysis.

UNIT 6: DIFFERENTIAL EQUATIONS ORDINARY DIFFERENTIAL EQUATIONS:

Second order homogeneous equations – Initial value problems – Linear dependence and independence – Formula for Wronskian - Non-homogeneous equations of order two - Homogeneous and non-homogeneous equations of order n – Annihilator method to solve a non - homogeneous equation – Initial value problems for the homogeneous equation – Solutions of the homogeneous equations – Wronskian and linear independence – Reduction of the order of a homogeneous equation – Linear equation with regular singular points – Euler equation - Second order

equations with regular singular points – Solutions and properties of Legendre and Bessel's equation – Equations with variables separated – Exact equations – Method of successive approximations - Lipschitz condition – Convergence of the successive approximations.

PARTIAL DIFFERENTIAL EQUATIONS:

Integral surfaces passing through a given curve – Surfaces orthogonal to a given system of surfaces – Compatible system of equations - Charpit's method – Classification of second order Partial Differential Equations – Reduction to canonical form - Adjoint operators - Riemann's method- One-dimensional wave equation – Initial value problem - D'Alembert's solution – Riemann – Volterra solution – Vibrating string - Variables Separable solution – Forced vibrations - Solutions of non-homogeneous equation – Vibration of a circular membrane – Diffusion equation - Solution of diffusion equation in cylindrical and spherical polar coordinates by method of Separation of variables - Solution of diffusion equation by Fourier transform – Boundary value problems – Properties of harmonic functions - Green's function for Laplace equation - The methods of images - The eigen function method.

UNIT 7: MECHANICS AND CONTINUM MECHANICS MECHANICS:

The Mechanical system - Generalized coordinates – Constraints - Virtual work – and Energy Momentum derivation of Lagrange's equations – Examples – Integrals of the motion Hamilton's principle - Hamilton's equations – Other variational principle - Hamilton principle function – Hamilton – Jacobi equation – Separability – Differential forms and generating functions – Special transformations – Lagrange and Poisson brackets.

CONTINUM MECHANICS:

Summation convention - Components of a tensor - Transpose of a tensor - Symmetric and anti-symmetric tensor – Principal values and directions - Scalar invariants - Material and spatial descriptions - Material derivative – Deformation – Principal strain – Rate of deformation - Conservation of mass – Compatibility conditions – Stress vector and tensor – Components of a stress tensor – Symmetry – Principal stresses – Equations of motion

- Boundary conditions –

Isotropic solid - Equations of infinitesimal theory – Examples of elastodynamics elastostatics –

Equations of hydrostatics - Newtonian fluid – Boundary conditions – Stream lines examples of laminar flows – Vorticity vector - Irrotational flow.

UNIT 8: MATHEMATICAL STATISTICS AND NUMERICAL METHODS

MATHEMATICAL STATISTICS:

Sampling distributions – Characteristics of good estimators – Method of moments – Maximum likelihood estimation – Interval estimates for mean, variance and proportions- Type I and type II errors – Tests based on Normal, t, and F distributions for testing of mean, variance and proportions - Tests for independence of attributes and goodness of fit – Method of least squares - Linear regression – Normal regression analysis- Normal correlation analysis - Partial and multiple correlation – Multiple linear regression – Analysis of variance - One-way and two-way classifications – Completely randomized design – Randomized block design – Latin square design – Covariance matrix - Correlation matrix – Normal density function – Principal components – Sample variation by principal components – Principal components by graphing.

NUMERICAL METHODS:

Direct methods : Gauss elimination method – Error analysis – Iterative methods : Gauss-Jacobi and Gauss-Seidel – Convergence considerations – Eigen value Problem : Power method - Interpolation: Lagrange's and Newton's interpolation – Errors in interpolation – Optimal points for interpolation – Numerical differentiation by finite differences - Numerical integration: Trapezoidal, Simpson's and Gaussian quadratures - Error in quadratures – Norms of functions – Best approximations: Least squares polynomial approximation – Approximation with Chebyshev polynomials – Piecewise linear and cubic Spline approximation - Single-step methods: Euler's method - Taylor series method – Runge – Kutta method of fourth order – Multistep methods : Adams-Bashforth and Milne's methods – Linear two point BVPs: Finite difference method-Elliptic equations: Five point finite difference formula in rectangular region - truncation error; One-dimensional parabolic equation: Explicit and Crank-Nicholson schemes; Stability of the above schemes - One-dimensional hyperbolic equation: Explicit scheme.

UNIT 9: DIFFERENTIAL GEOMETRY AND GRAPH THEORY DIFFERENTIAL GEOMETRY:

Representation of space curves - Unique parametric representation of a space curve - Arc-length – Tangent and osculating plane – Principal normal and bi-normal- Curvature and torsion – Behaviour of a curve near one of its points – The curvature and torsion of a curve as the intersection of two surfaces – Contact between curves and surfaces – Osculating circle and Osculating sphere – Locus of centres of spherical curvature – Tangent surfaces, involutes and evolutes – Intrinsic equations of space curves – Fundamental existence theorem – Helices – Definition of a surface - Nature of points on a surface – Representation of a surface – Curves on surfaces – Tangent plane and surface normal – The general surfaces of revolution – Helicoids – Metric on a surface - Direction coefficients on a surface – Families of curves – Orthogonal trajectories - Double family of curves – Isometric correspondence – Intrinsic properties - Geodesics and their differential equations – Canonical geodesic equations – Geodesics on surface revolution – Normal property of geodesics – Differential equations of geodesics using normal property – Existence theorems – Geodesic parallels – Geodesic curvature - Gauss – Bonnet theorem – Gaussain curvature - Surfaces of constant curvature.

GRAPH THEORY:

Graphs and subgraphs: Graphs and simple graphs – Graph isomorphism - Incidence and adjacency matrices – Subgraphs – Vertex degrees – Path and Connection cycles – Applications : The shortest path problem - Trees: Trees – Cut edges and bonds - Cut vertices - Cayley's formula – Connectivity : Connectivity – Blocks – Euler tours and Hamilton cycles: Euler tours - Hamilton cycles - Applications: The Chinese postman problem – Matchings : Matchings – Matching and coverings in bipartite graphs - Perfect matchings – Edge colourings : Edge chromatic number - Vizing's theorem - Applications: The timetabling problem - Independent sets and cliques : Independent sets-Ramsey's theorem - Turan's theorem – Vertex colourings : Chromatic number - Brook's theorem - Hajos' conjecture – Chromatic polynomials - Girth and chromatic number – Planar graphs : Plane and planar graphs – Dual graphs - Euler's formula – Bridges - Kuratowski's Theorem - The Five color theorem and the four color conjecture – Non Hamiltonian planar graphs.

UNIT-10: MATHEMATICAL PROGRAMMING AND FLUID DYNAMICS

MATHEMATICAL PROGRAMMING:

Linear programming : Formulation and graphical solutions – Simplex method – Transportation and Assignment problems – Advanced linear programming : Duality - Dual simplex method – Revised simplex method - Bounded variable technique - Integer programming : Cutting plane algorithm – Branch and bound technique – Applications of integer programming – Non-linear programming: Classical optimization theory Unconstrained problems - Constrained problems - Quadratic programming - Dynamic programming : Principle of optimality – Forward and backward recursive equations – Deterministic dynamic programming applications.

FLUID DYNAMICS:

Kinematics of fluids in motion : Real and ideal fluids – Velocity – Acceleration – Streamlines - Pathlines - Steady and unsteady flows – Velocity potential - Vorticity vector - Local and particle rates of change - Equation of continuity – Conditions at a rigid boundary – Equations of motion of a fluid : Pressure at a point in a fluid – Boundary conditions of two inviscid immiscible fluids - Euler's equations of motion - Bernoullt's equation - Some potential theorems – Flows involving axial symmetry – Two dimensional flows : Two-dimensional flows – Use of cylindrical polar co-ordinates - Stream function, complex potential for two-dimensional flows, irrotational, incompressible flow – Complex potential for standard two-dimensional flows – Two dimensional image systems – Milne – Thomson circle theorem - Theorem of Blasius – Conformal transformation and its applications : Use of conformal transformations - Hydro-dynamical aspects of conformal mapping – Schwarz Christoffel transformation – Vortex rows – Viscous flows : Stress - Rate of strain – Stress analysis – Relation between stress and rate of strain-Cofficient of viscosity - Laminar flow – Navier – Stokes equations of motion – Some problems in viscous flow.